Summer Work
7th → Algebra
2023

Show all work!

(Use a separate sheet of paper if necessary. Any additional papers showing work should be stapled to this packet)

Due: First Day of School

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Integers I

Hints/Guide:

To add integers with the same sign (both positive or both negative), add their absolute values and use the same sign. To add integers of opposite signs, find the difference of their absolute values and then take the sign of the larger absolute value.

To subtract integers, add its additive inverse. For example, $6 - 11 = 6 + -11 = -5$

Exercises: Solve the following problems. **Show work** **NO calculators.**

1. $(-4) + (-5) =$
2. $-9 - (-2) =$
3. $6 - (-9) =$
4. $(-6) - 7 =$
5. $7 - (-9) =$
6. $15 - 24 =$
7. $(-5) + (-8) =$
8. $-15 + 8 - 8 =$
9. $14 + (-4) - 8 =$
10. $14.5 - 29 =$
11. $-7 - 6.85 =$
12. $-8.4 - (-19.5) =$
13. $29 - 16 + (-5) =$
14. $-15 + 8 - (-19.7) =$
15. $45.6 - (-13.5) + (-14) =$
16. $-15.98 - 6.98 - 9 =$
17. $-7.24 + (-6.28) - 7.3 =$
18. $29.45 - 56.009 - 78.2 =$
19. $17.002 + (-7) - (-5.23) =$
20. $45.9 - (-9.2) + 5 =$

Integers II

Hints/Guide:

The rules for multiplying integers are:

Positive · Positive = Positive
Negative · Negative = Positive
Positive · Negative = Negative
Negative · Positive = Negative

The rules for dividing integers are the same as multiplying integers

Exercises: Solve the following problems. **Show Work and No Calculators**

1. $4 \cdot (-3) \cdot 6 =$
2. $5\ (-12) \cdot (-4) =$
3. $(4)(-2)(-3) =$
4. $\frac{(-5)(-6)}{-2} =$
5. $\frac{6(-4)}{8} =$
6. $\frac{-56}{2^3} =$
7. $6\ (-5 - (-6)) =$
8. $8\ (-4 - 6) =$
9. $-6\ (9 - 11) =$
10. $\frac{-14}{2} + 7 =$
11. $8 - \frac{-15}{-3} =$
12. $-3 + \frac{-12 \bullet (-5)}{4} =$
13. $\frac{-6 - (-8)}{-2} =$
14. $-7 + \frac{4 + (-6)}{-2} =$
15. $45 - 14\ (5 - (-3)) =$
16. $(-4 + 7)\ (-16 + 3) =$
17. $16 - (-13)\ (-7 + 5) =$
18. $\frac{4 + (-6) - 5 - 3}{-6 + 4} =$
19. $(-2)^3\ (-5-(-6)) =$
20. $13\ (-9 + 17) + 24 =$

Rename Fractions, Percents, and Decimals

Hints/Guide:

To convert fractions into decimals, we start with a fraction, such as $\frac{3}{5}$, and divide the numerator (the top number of the fraction) by the denominator (the bottom number of the fraction). So:

$$5\overline{)3.0}^{\,.6} \qquad \text{and the fraction } \frac{3}{5} \text{ is equivalent to the decimal 0.6}$$

To convert a decimal to a percent, we multiply the decimal by 100 (percent means a ratio of a number compared to 100). A short-cut is sometimes used of moving the decimal point two places to the right (which is equivalent to multiplying a number by 100), so $0.6 \cdot 100 = 60$ and $\frac{3}{5} = 0.6 = 60\%$.

To convert a percent to a decimal, we divide the percent by 100,
60% is the same as $60 \div 100$, which is 0.6, so $60\% = 0.6$

To convert a fraction into a percent, we can use proportions to solve, so
$\frac{3}{5} = \frac{x}{100}$ and using cross products to solve, $5x = 300$ or $x = 60\%$

Exercises: Complete the chart

	Fraction	Decimal	Percent
1.		0.04	
2.			125%
3.	$\frac{2}{3}$		
4.		1.7	
5.			0.6%
6.	$3\frac{1}{2}$		
7.		0.9	
8.			70%
9.	$\frac{17}{25}$		
10.		0.007	

Squares and Square Roots

Hints/Guide:

Exponents are a way to represent repeated multiplication, so that 3^4 means 3 multiplied four times, or $3 \cdot 3 \cdot 3 \cdot 3$, which equals 81. In this example, 3 is the base and 4 is the power.

Roots are the base numbers that correspond to a given power, so the square (referring to the power of 2) root of 81 is 9 because $9 \cdot 9 = 81$ and the fourth root of 81 is 3 because $3 \cdot 3 \cdot 3 \cdot 3$ is 81.

$\sqrt[n]{x}$, where n is the root index and x is the radicand

*****You can use a calculator but try to evaluate by hand first.**

Exercises: Evaluate. ~~**NO CALCULATORS**~~ **(Show work when there is more than one step)**

1. $3^4 =$
2. $(-4)^3 =$
3. $-4^3 =$
4. $\sqrt{49} =$
5. $\sqrt{289} =$
6. $\sqrt[3]{64} =$
7. $(8-4)^2 =$
8. $(4-2)^2 (5-8)^3 =$
9. $5(8-3)^2 =$
10. $\sqrt{25-16} =$
11. $\sqrt{5(9 \bullet 125)} =$
12. $\sqrt{(8-4)(1+3)} =$
13. $\sqrt{45-4(3+6)} =$
14. $-\sqrt{14(16-2)} =$

Find Percent of a Number

Hints/Guide:

To determine the percent of a number, we must first convert the percent into a decimal by dividing by 100 (which can be short-cut by moving the decimal point in the percentage two places to the left), then multiplying the decimal by the number. For example:

4.5% of $240 = 4.5\% \cdot 240 = 0.045 \cdot 240 = 10.8$

Exercises: Solve for n.

SHOW ALL WORK. Use a separate sheet of paper (if needed) and staple to this page.

1. 305% of $450 = n$
2. 7.5% of $42 = n$
3. 120% of $321 = n$
4. 15% of $54 = n$
5. 0.65% of $320 = n$
6. 800% of $64 = n$
7. 95% of $568 = n$
8. 150% of $38 = n$
9. 215% of $348 = n$
10. 85% of $488 = n$
11. 9.05% of $750 = n$
12. 160% of $42 = n$
13. 60% of $78 = n$
14. 0.4% of $480 = n$
15. 0.10% of $435 = n$
16. 2.4% of $54 = n$

Solve Problems Using Percents

Hints/Guide:

When solving percent problems, we apply the rules for finding percent of a number in realistic situations. For example, to find the amount of sales tax on a $450.00 item if the tax rate is 5%, we find 5% of 450 ($.05 \cdot 450 = 22.5$), and then label our answer in dollars, getting $22.50.

Exercises: Solve the following items ~~**without your calculator.**~~

SHOW ALL WORK. Use a separate sheet of paper (if needed) and staple to this page.

1. Susie has just bought a pair of jeans for $49.95, a sweater for $24.50, and a jacket for $85.95. The sales tax is 5%. What is her total bill?

2. Jack bought a set of golf clubs for $254.00 and received a rebate of 24%. How much was the rebate?

3. A construction manager calculates it will cost $2,894.50 for materials for her next project. She must add in 12.5% for scrap and extras. What will be the total cost?

4. The regular price for a video game system is $164.50 but is on sale for 30% off. What is the amount of the discount?

   What is the sale price?

5. Cindy earns a 15% commission on all sales. On Saturday, she sold $985.40 worth of merchandise. What was the amount of commission she earned on Saturday?

6. The band had a fundraiser and sold $25,800 worth of candy. They received 38% of this amount for themselves. How much did they receive?

Evaluating Algebraic Expressions

1. Substitute the given values for the variables in the expression
2. Evaluate the expression using the order of operations
   - Parentheses/Brackets (inside to outside)
   - Exponents
   - Multiplication/Division (left to right)
   - Addition/Subtraction (left to right)

ex: $9x^2 - 4(y + 3z)$
for $x = -3, y = 2, z = 5$

$9(-3)^2 - 4(2 + 3 \cdot 5)$

$9(-3)^2 - 4(2 + 15)$

$9(-3)^2 - 4 \cdot 17$

$9 \cdot 9 - 4 \cdot 17$

$81 - 4 \cdot 17$

$81 - 68 = \boxed{13}$

The Distributive Property

1. Multiply the number outside the parentheses by each term in the parentheses.
2. Keep the addition/subtraction sign between each term.

ex: $5(8x - 3)$

$5(8x - 3)$

$5(8x) - 5(3)$

$\boxed{40x - 15}$

Simplifying Algebraic Expressions

1. Clear any parentheses using the Distributive Property
2. Add or subtract like terms (use the sign in front of each term to determine whether to add or subtract)

ex: $2(3x - 4) - 12x + 9$

$2(3x - 4) - 12x + 9$

$6x - 8 - 12x + 9$

$\boxed{-6x + 1}$

Evaluate each expression for $a = 9$, $b = -3$, $c = -2$, $d = 7$. Show your work. use parentheses when plugging in

1. $a - cd$	2. $2b^3 + c^2$	3. $\frac{a + d - c}{b}$	4. $(a - b)^2 + d(a + c)$
5. $4c - (b - a)$	6. $\frac{a}{b} - 5a$	7. $2bc + d(12 - 5)$	8. $b + 0.5[8 - (2c + a)]$

Simplify each expression using the Distributive Property.

9. $5(2g - 8)$	10. $7(y + 3)$	11. $-3(4w - 3)$	12. $(6r + 3)2$

Simplify each expression, showing all work.

13. $8(x + 1) - 12x$	14. $6w - 7 + 12w - 3z$	15. $9n - 8 + 3(2n - 11)$	16. $3(7x + 4y) - 2(2x + y)$
17. $(15 + 8d)(-5) - 24d + d$	18. $9(b - 1) - c + 3b + c$	19. $20f - 4(5f + 4) + 16$	20. $8(h - 4) - h - (h + 7)$

The Distributive Property

The Distributive Property states for any number *a, b,* and *c*:

1. $a(b+c)=ab+ac$ or $(b+c)a=ba+ca$
2. $a(b-c)=ab-ac$ or $(b-c)a=ba-ca$

Practice: Rewrite each expression using the distributive property.

1. $7(h-3)$

2. $-3(2x+5)$

3. $(5x-9)4$

4. $\frac{1}{2}(14-6y)$

5. $3(7x^2-3x+2)$

6. $\frac{1}{4}(16x-12y+4z)$

7. $(9-2x+3xy)\cdot -4$

8. $0.3(40a+10b-5)$

Combining Like terms

Terms in algebra are numbers, variables or the product of numbers and variables. In algebraic expressions terms are separated by addition (+) or subtraction (-) symbols. Terms can be combined using addition and subtraction if they are **like-terms**.

Like-terms have the same variables to the same power.

Example of like-terms: $5x^2$ and $-6x^2$

Example of terms that are **NOT** like-terms: $9x^2$ and $15x$
*Although both terms have the variable **x**, they are not being raised to the same power*

To combine like-terms using addition and subtraction, add or subtract the numerical factor

Example: Simplify the expression by combining like-terms

$$\begin{aligned} 8x^2+9x-12x+7x^2 &= (8+7)x^2+(9-12)x \\ &= 15x^2+-3x \\ &= 15x^2-3x \end{aligned}$$

Practice: Simplify each expression

1. $5x-9x+2$

2. $3q^2+q-q^2$

3. $c^2+4d^2-7d^2$

4. $5x^2+6x-12x^2-9x+2$

5. $2(3x-4y)+5(x+3y)$

6. $10xy-4(xy+2x^2y)$

Solving One-Step Equations

1. Cancel out the number on the same side of the equal sign as the variable using inverse operations (addition/subtraction; multiplication/division)
2. Be sure to do the same thing to both sides of the equation!

ex: $-18 = 6j$

$\frac{-18}{6} = \frac{6j}{6}$

$-3 = j \rightarrow \boxed{j = -3}$

Solving Two-Step Equations

1. Undo operations one at a time with inverse operations, using the order of operations in reverse (i.e. undo addition/subtraction before multiplication/division)
2. Be sure to always do the same thing to both sides of the equation!

ex: $\frac{a}{7} - 12 = -9$

$\frac{a}{7} - 12 = -9$

$+12 \quad +12$

$7 \times \frac{a}{7} = 3 \times 7$

$\boxed{a = 21}$

Solving Multi-Step Equations

1. Clear any parentheses using the Distributive Property
2. Combine like terms on each side of the equal sign
3. Get the variable terms on the same side of the equation by adding/subtracting a variable term to/from both sides of the equation to cancel it out on one side
4. The equation is now a two-step equation, so finish solving it as described above

ex: $5(2x - 1) = 3x + 4x - 1$

$10x - 5 = 3x + 4x - 1$

$10x - 5 = 7x - 1$

$-7x \quad -7x$

$3x - 5 = -1$

$+5 \quad +5$

$\frac{3x}{3} = \frac{4}{3}$

$\boxed{x = \frac{4}{3}}$

Solving Equations II

Hints/Guide:

The key in equation solving is to isolate the variable, to get the letter by itself. In two-step equations, we must undo addition and subtraction first, then multiplication and division. Remember the golden rule of equation solving: If we do something to one side of the equation, we must do the exact same thing to the other side. Examples:

1. $4x - 6 = -14$

$+6 \quad +6$

$\underline{4x} = \underline{-8}$

$4 \qquad 4$

$x = -2$

Solve: $4(-2) - 6 = -14$

$-8 - 6 = -14$

$-14 = -14$

2. $\frac{x}{-6} - 4 = -8$

$+4 \quad +4$

$-6 \cdot \frac{x}{-6} = -4 \cdot -6$

$x = 24$

Solve: $(24/-6) - 4 = -8$

$-4 - 4 = -8$

$-8 = -8$

Exercises: Solve the following problems: ~~**No Calculators!**~~

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1. $-4t - 6 = 22$
2. $\frac{m}{-5} + 6 = -4$
3. $-4r + 5 = -25$
4. $\frac{x}{-3} + (-7) = 6$
5. $5g + (-3) = -12$
6. $\frac{y}{-2} + (-4) = 8$

Solving Equations III

Hints/Guide:

When solving equations that include basic mathematical operations, we must simplify the mathematics first, then solve the equations. For example:

$$5(4-3)+7x=4(9-6)$$
$$5(1)+7x=4(3)$$
$$5+7x=12$$
$$-5 \quad -5$$
$$\frac{7x}{7}=\frac{7}{7}$$
$$x=1$$

Check: $5(4-3)+7(1)=4(9-6)$
$5+7=4(3)$
$12=12$

Exercises: Solve the following equations using the rules listed on the previous pages:
SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

1. $4x+8-6=2(9-2)$
2. $\frac{t}{5}-7+31=8(6-4)$
3. $5(t-4)=9(7-3)$
4. $9-5(4-3)=-16+\frac{x}{3}$
5. $6t-9-3t=8(7-4)$
6. $7(6-(-8))=\frac{t}{-4}+2$
7. $7(3-6)=6(4+t)$
8. $4r+5r-6r=15+6$
9. $3(5+x)=5(7-(-2))$

Equations - Variables on Each Side

Hints/Guide:

As we know, the key in equation solving is to isolate the variable. In equations with variables on each side of the equation, we must combine the variables first by adding or subtracting the amount of one variable on each side of the equation to have a variable term on one side of the equation. Then, we must undo the addition and subtraction, then multiplication and division. Remember the golden rule of equation solving. Examples:

$$\begin{array}{r} 8x - 6 = 4x + 5 \\ -4x \quad\quad -4x \\ 4x - 6 = \quad 5 \\ +6 \quad\quad +6 \\ \frac{4x}{4} = \frac{11}{4} \end{array}$$

$$x = 2\frac{3}{4}$$

$$\begin{array}{r} 5 - 6t = 24 + 4t \\ +6t \quad\quad +6t \\ 5 \quad = 24 + 10t \\ -24 \quad\quad -24 \\ \frac{-19}{10} = \frac{10t}{10} \end{array}$$

$$-1\frac{9}{10} = t$$

Exercises: Solve the following problems: ~~**No Calculators!**~~

SHOW ALL WORK. Use a separate sheet of paper (if necessary) and staple to this page.

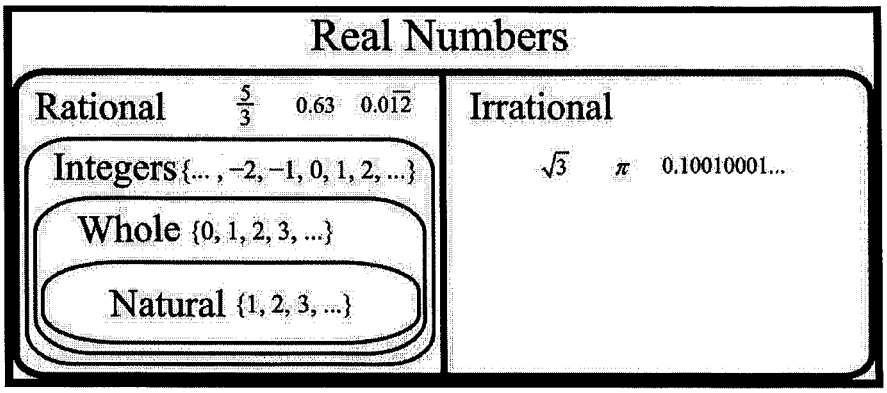
1. $4r - 7 = 8r + 13$
2. $14 + 3t = 5t - 12$
3. $4x + 5 = 3x - 3$

4. $6y + 5 = 4y - 13$
5. $5x - 8 = 6 - 2x$
6. $7p - 8 = -4p + 6$

Solve each equation, showing all work.

Try ones w/ a ★. They are more challenging

21. $f - 64 = -23$	22. $-7 = 2d$	23. $\frac{b}{-12} = -6$	24. $13 = m + 21$
25. $5x - 3 = -28$	26. $\frac{w + 8}{-3} = -9$	27. $-8 + \frac{h}{4} = 13$	28. $22 = 6y + 7$
★29. $8x - 4 = 3x + 1$	30. $-2(5d - 8) = 20$	★31. $7r + 21 = 49r$	★32. $-9g - 3 = -3(3g + 2)$
★33. $5(3x - 2) = 5(4x + 1)$	★34. $3d - 4 + d = 8d - (-12)$	★35. $f - 6 = -2f + 3(f - 2)$	★36. $-2(y - 1) = 4y - (y + 2)$



The **Real** number system is made up of two main sub-groups **Rational numbers** and **Irrational numbers.**

The set of rational numbers includes several subsets: **natural numbers, whole numbers, and integers.**

- **Real Numbers-** any number that can be represented on a number-line.
  - **Rational Numbers-** a number that can be written as the ratio of two integers (this includes decimals that have a definite end or repeating pattern)

    Examples: $2, -5, \frac{-3}{2}, \frac{1}{3}, 0.253, 0.\overline{3}$
    - **Integers-** positive and negative whole numbers and 0
      Examples: $-5, -3, 0, 8$ ...
    - **Whole Numbers** – the counting numbers from 0 to infinity
      Examples: { 0, 1, 2, 3, 4, ...}
    - **Natural Numbers-** the counting numbers from 1 to infinity
      Examples: { 1, 2, 3, 4... }
  - **Irrational Numbers-** Non-terminating, non-repeating decimals (including π, and the square root of any number that is not a perfect square.)

    Examples: 2π, $\sqrt{3}$, $\sqrt{23}$, 3.21211211121111....

Practice: Name all the sets to which each number belongs.

1. -4.2 _____
2. $3\sqrt{5}$ _____
3. $\frac{5}{3}$ _____
4. 9 _____
5. $\sqrt{16}$ _____
6. $-\frac{8}{2}$ _____

Algebraic Translations

Hints/Guide:

Key Words for Translations:

Add	Subtract	Multiply	Divide	Inequalities	Variable	=
Plus Sum Longer Than Greater Than Together Total Increased More Than In all And	Decreased Smaller Less than Difference Reduced Differ Fewer Shorter Than Minus Diminished	Per For Every For each Triple Multiplied Of Times Twice Double	One-third Quotient Divided by Each part Half as much Spilt equally	< is less than > is greater than ≤ is less than or equal to ≥ is greater than or equal to	a number some number quantity	Same as Equals Is Total Was Result Outcome Answer

Practice: Translate each phrase into a mathematical statement

1. Seven plus five times a number is greater than or equal to -9

2. Eight times a number increased by 6 is 62

3. One half of a number is equal to 14

4. 6 less than 8 times some number

5. a number divided by 9

6. *p* decreased by 5

7. twice a number decreased by 15 is equal to -27

8. 9 less than 7 times some number is -6

9. the sum of a number and eight is less than 2

10. eleven increased by a number is -12

Word Problems

Hints/Guide:

Translate each word problem into an algebraic equation, using x for the unknown, and solve. Write a "let x =" for each unknown; write an equation; solve the equation; substitute the value for x into the let statements(s) to answer the question.

For Example:

Kara is going to Maui on vacation. She paid $325 for her plane ticket and is spending $125 each night for the hotel. How many nights can she stay in Maui if she has $1200?

Step 1: What are you asked to fine? Let variables represent what you are asked to find.

How many nights can Kara stay in Maui?

Let x = The number of nights Kara can stay in Maui

Step 2: Write an equation to represent the relationship in the problem.

$325 + 125x = 1200$

Step 3: Solve the equation for the unknown

$$\begin{aligned} 325 + 125x &= 1200 \\ -325 \quad & \quad -325 \\ 125x &= 875 \\ x &= 7 \end{aligned}$$

Kara can spend 7 nights in Maui

Word Problem Practice Set

1. A video store charges a one-time membership fee of $12.00 plus $1.50 per video rental. How many videos can Stewart rent if he spends $21?

2. Bicycle city makes custom bicycles. They charge $160 plus $80 for each day that it takes to build the bicycle. If you have $480 to spend on your new bicycle, how many days can it take Bicycle City to build the bike?

3. Darel went to the mall and spent $41. He bought several t-shirts that each cost $12 and he bought 1 pair of socks for $5. How many t-shirts did Darel buy?

Inequalities

Hints/Guide:

In solving inequalities, the solution process is very similar to solving equalities. The goal is still to isolate the variable, to get the letter by itself. However, the one difference between equations and inequalities is that when solving inequalities, when we multiply or divide by a negative number, we must change the direction of the inequality. Also, since an inequality has many solutions, we can represent the solution of an inequality by a set of numbers or by the numbers on a number line.

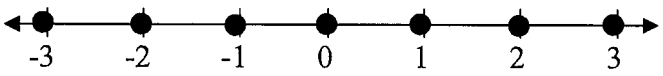
Inequality - a statement containing one of the following symbols:

$<$ is less than $>$ is greater than $\leq$ is less than or equal to

$\geq$ is greater than or equal to $\neq$ is not equal to

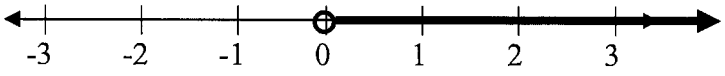
Examples:

1. Integers between -4 and 4.



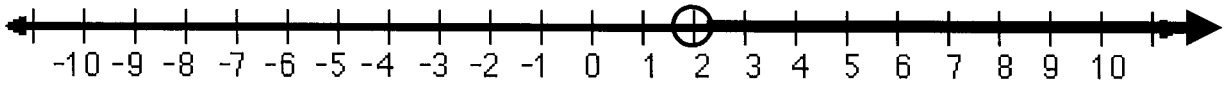
2. All numbers between -4 and 4.

3. The positive numbers.



So, to solve the inequality $-4x < -8$ becomes $\frac{-4x}{-4} < \frac{-8}{-4}$

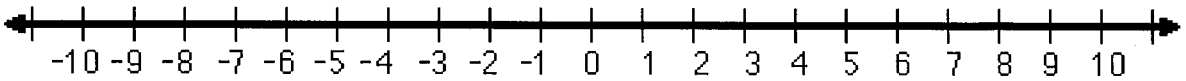
and therefore $x > 2$ is the solution (this is because whenever we multiply or divide an inequality by a negative number, the direction of the inequality must change) and can be represented as:



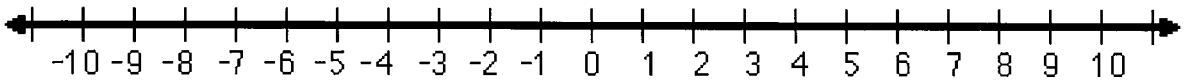
Exercises: Solve the following problems: **No Calculators!**

1. $4x > 9$

2. $-5t \geq -15$



3. $\frac{x}{2} \geq 3$



4. $\frac{x}{-4} > 2$

Laws of Exponents

Hints/Guide:

There are certain rules when dealing with exponents that we can use to simplify problems. They are:

Adding powers	$a^m a^n = a^{m+n}$
Multiplying powers	$(a^m)^n = a^{mn}$
Subtracting powers	$\frac{a^m}{a^n} = a^{m-n}$
Negative powers	$a^{-n} = \frac{1}{a^n}$
To the zero power	$a^0 = 1$

Here are some examples of problems simplified using the above powers:

$4^3 \bullet 5^5 = 4^8$ $\quad (4^3)^3 = 4^9$ $\quad 4^5 \div 4^3 = 4^2$ $\quad 4^{-4} = \frac{1}{4^4} = \frac{1}{256}$ $\quad 4^0 = 1$

Exercises: Simplify the following problems using exponents (Do not multiply out).

1. $5^2 5^4 =$
2. $7^{-3} 7^5 =$
3. $(12^4)^3 =$
4. $(6^5)^2 =$
5. $5^9 \div 5^4 =$
6. $10^3 \div 10^{-5} =$
7. $7^{-3} =$
8. $3^{-4} =$
9. $124^0 =$
10. $-9^0 =$
11. $(3^5 \bullet 3^2)^3 =$
12. $5^3 \bullet 5^4 \div 5^7 =$