

AP Calculus Summer Assignment

Name _____

Welcome to AP Calculus! This packet contains a set of problems that serve a pre-requisite for AP Calculus. These skills are extremely important. Many times you will find that the calculus is actually the easy part, and the algebra is more challenging. This packet will help you to maintain and improve your skills.

Please follow the instructions in each section, and show all work supporting your answer. Most sections require that you solve the problem without the use of a calculator. Please ensure to have this packet completed prior to the first week of school.

The AP Calculus curriculum and pace necessitates that you are comfortable with the topics in this packet, and you will have ample opportunity in class to continually engage with these topics as we explore new concepts. I look forward to working with you in AP Calculus next year!!

Topical Understandings Covered in this packet:

- Simplifying algebraic expressions
- Determining vertical and horizontal asymptotes
- Exponential and logarithmic functions
- Unit Circle and exact values
- Trigonometric identities
- Long Division and Synthetic Division
- Completing the Square
- Limits: graphically, tabular, algebraic techniques
- Derivatives: definition; average rate of changes vs. instantaneous;

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Simplify the following algebraic expressions. Where appropriate, express your final solution utilizing only positive exponents.

1. $-5 \left(\frac{3}{2}\right) (4 - 9x)^{-\frac{1}{2}} (-9)$

2. $2 \left(\frac{2}{2-x}\right) \left[\frac{-2}{(2-x)^2}\right]$

3. $\frac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$

4. $\left(\frac{1}{x^{-2}} + \frac{4}{x^{-1}y^{-1}} + \frac{1}{y^{-2}}\right)^{-1/2}$

5. $\frac{2x - \frac{13x-3}{x-4}}{2x + \frac{x+3}{x-4}}$

6. $\frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$

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Identify the horizontal and vertical asymptotes, if they exist, for the following:

$$7. f(x) = \frac{x}{x^2 - 4x + 4}$$

$$8. g(x) = \frac{4x - 4}{x^3 - 8}$$

Solve the exponential or log equations:

$$9. 12^{m+3} = \frac{1^{2m}}{3}$$

$$10. \log_5(x + 1) - \log_5 4 = \log_5 6$$

Use the properties of logs to re-write the following equations (do not evaluate the expression):

$$11. \log_b \frac{5x^3y}{\sqrt[4]{2z}}$$

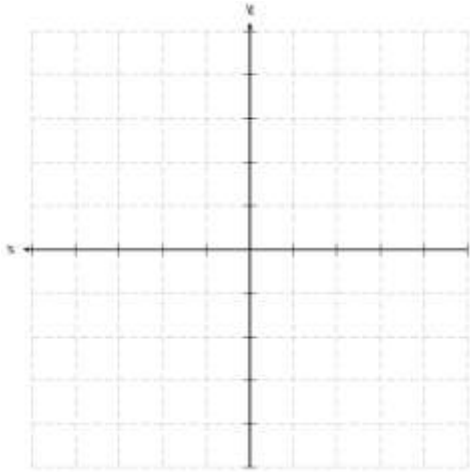
$$12. \log_b \sqrt{\frac{13(x+5)}{(2x-3)^3}}$$

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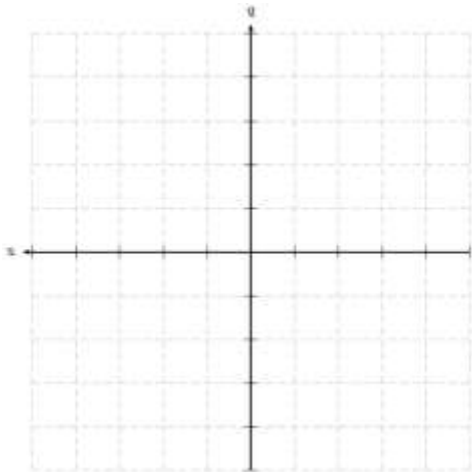
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Find the exact value of each trigonometric function. Draw the graph in standard position.

13. $\tan 420^\circ$



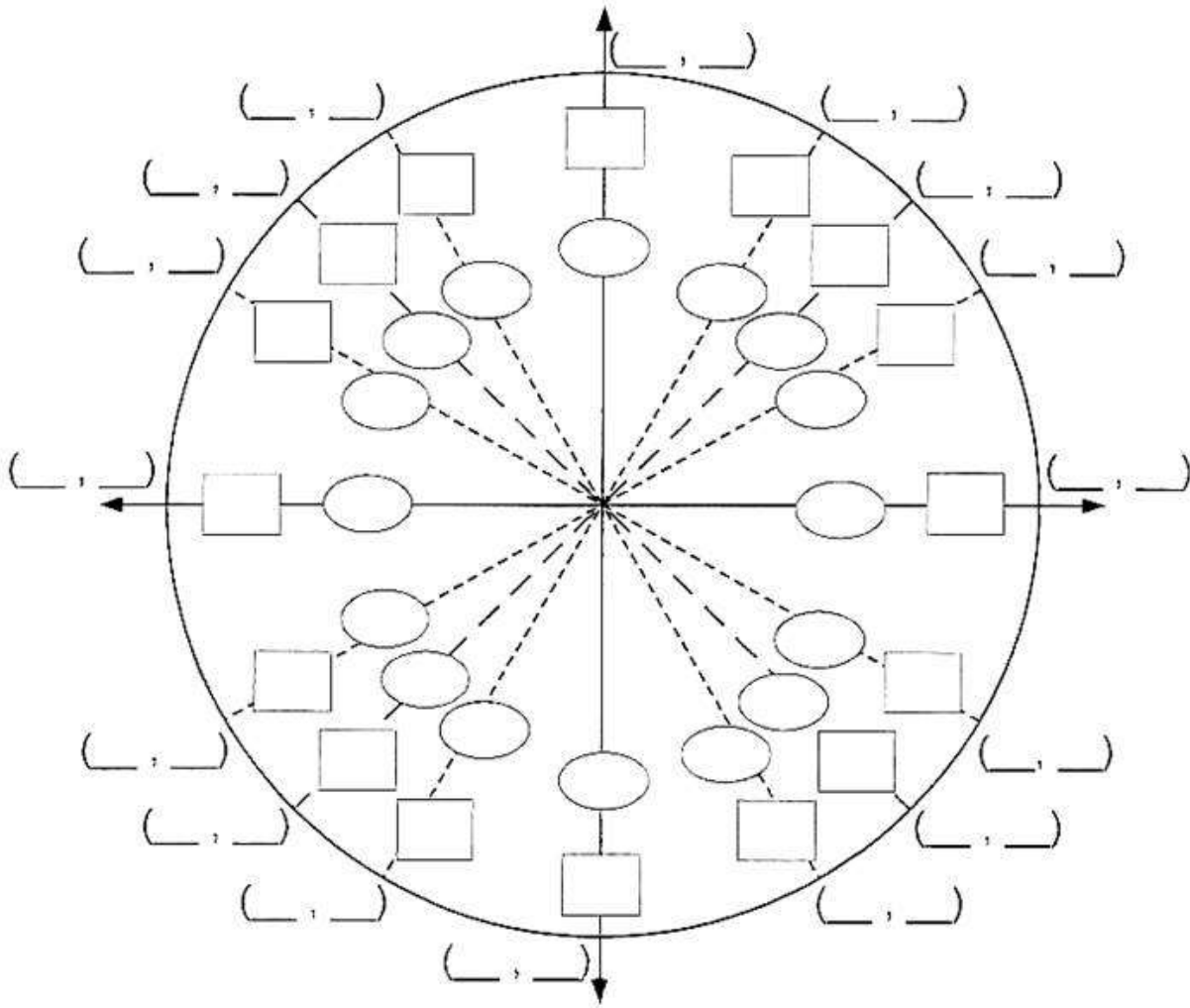
14. $\csc(-225^\circ)$



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15. Identify all parts of the Unit Circle, including degree, radian, and coordinates of each point (exact values):



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Identify the amplitude, period, midline, maximum and minimum values and the zeros of the function in the interval $[-2\pi, 2\pi]$. Use your understanding of transformations, not your calculator.

$$16. y = 3\sin\left(2\theta + \frac{5\pi}{6}\right) + 2$$

$$17. y = 4\cos\left(4\theta + \frac{2\pi}{3}\right)$$

Simplify the following expression to either a constant or a basic trigonometric function.

$$18. \quad \frac{\sec^2 x - \tan^2 x}{\cos^2 x + \sin^2 x}$$

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Use the basic identities to change the expression to one involving sine and/or cosine. Then simplify to a basic trigonometric function.

19. $\frac{\sec^2 x \csc x}{\sec^2 x + \csc^2 x}$

Prove the Identity:

20. $\frac{\sin x - \cos x}{\sin x + \cos x} = \frac{2 \sin^2 x - 1}{1 + 2 \sin x \cos x}$

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Polynomial Division and Factor theorems

Divide the following polynomials utilizing long division.

21. $(9x^5 - 3x^3 + 21x^2 - 2x + 4) \div (3x^2 + 1)$

22. $(2x^2 - 7x + 4) \div (x - 2)$

23. $(y^2 - 9) \div (y + 1)$

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Use synthetic division to divide the following polynomials:

24. If $(x + 1)$ is a factor of $2x^5 + 2x^4 - 5x^3 - 5x^2 - 3x - 3$, what are all the factors of $f(x)$.

25. $(x^2 + 3x - 3) \div (x - 3)$

26. $(7x^6 + 50x^3 + 20x) \div (x + 2)$

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Factor the following utilizing Completing the Square. Final answers are to be in the vertex form of a quadratic function which is $y = a(x - h)^2 + k$, where (h, k) is the vertex of the graph of the function.

27. $x^2 - 10x + 7 = 0$

28. $x^2 - 22x + 16 = 0$

29. $2x^2 - 20x + 60 = 0$

30. $3x^2 + 12x + 15 = 0$

31. $4x^2 + 32x = -68$

32. $-3x^2 - 18x - 6 = 0$

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33. $y = x^2 + 6x + 4$

34. $y = x^2 - 2x - 6$

Evaluate the following limits. If the limit is undefined or indeterminate, state so. Determine if an alternate algebraic technique can be utilized to determine if the limit exists, before stating DNE.

35. $f(x) = \begin{cases} x^2 + 1 & \text{if } x < 1 \\ \sqrt{x + 1} & \text{if } x \geq 1 \end{cases}$

Compare the following limits or state that they do not exist:

a. $\lim_{x \rightarrow 1^+} f(x):$

b. $\lim_{x \rightarrow 1^-} f(x):$

c. $\lim_{x \rightarrow 1} f(x):$

36. $\lim_{x \rightarrow -1} \frac{(2x-1)^2 - 9}{x+1}$

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$$37. \quad \lim_{x \rightarrow 0} \frac{\frac{1}{5+x} - \frac{1}{5}}{x}$$

$$38. \quad \lim_{x \rightarrow 1} \frac{\sqrt{x+3} - 2}{x-1}$$

$$39. \quad \lim_{x \rightarrow 0} \frac{6 \sin 2x}{3 \sin 7x} + \lim_{x \rightarrow 0} \frac{8 \sin 2x}{x} =$$

$$40. \quad \lim_{x \rightarrow 0} \pi^2$$

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$$41. \lim_{x \rightarrow \infty} \left(\frac{10x^2 + 25x + 1}{x^4 - 8} \right)$$

Utilizing the definition of the derivative, $m_{tan} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$, find the derivative of the following:

$$42. \quad f(x) = x^3 - x^2$$

Find the derivative of the following (Show All Work):

$$43. \quad f(x) = (x^2 + 8x - 4)(2x^{-2} + x^{-4})$$

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44. $f(x) = (x + 1)^{10}$

45. $f(x) = \frac{4x^8 - \sqrt{x}}{8x^4}$

46. $\frac{d}{dx} \csc(x^3 - 5x)$

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47. $\frac{d}{dx} \cos(\sqrt{3x})$

48. $y = \frac{1}{(x^2-x)^5}$