SUMMER 2025

Name: ________________

Ms. Francess

Algebra 1

Happy Summer!

In the contents of this binder you will find everything you will need to get yourself ready for Algebra 1.

There are 5 lesson videos to watch. Each is around 20 minutes.

On the following page, there are QR Codes that you may scan to watch the videos. You must use a device from home to access the videos (not your Chromebook). If you are having trouble accessing the videos, please email me.

With each video comes a notes page to follow along with.

For your summer work, you must complete and fill out all 5 note pages – one for each lesson. Then, you must fill out a questionnaire for each lesson.

*The questionnaires are **mandatory** but your grade will not suffer if you are struggling to understand, so fill them out **honestly**.*

Please keep all contents together and bring them with you on the first day of school in September. **This will count as your first test score of 8th grade.**

As always, feel free to reach out with any questions or concerns. I hope you all enjoy the summer!

–Ms. Francess

Summer Math Lesson Schedule 2025

Week	Title	Minutes	QR Code
1	Slope and Similarity	27	
2	Equations of Lines	24	
3	Finding the Slope of a Line	21	
4	Average Rate of Change	24	
5	Linear Functions	21	

Each lesson should be followed up by filling out the questionnaire (honestly) on the page after each lesson.

It is important for me to know where you struggled, so I can assist you better when we revisit these topics on a higher level during the school year.

All of the above topics will be covered in Algebra 1, which is why I want to expose the material to you over the summer.

Name: ______________________________ Date: ____________________

SLOPE AND SIMILARITY
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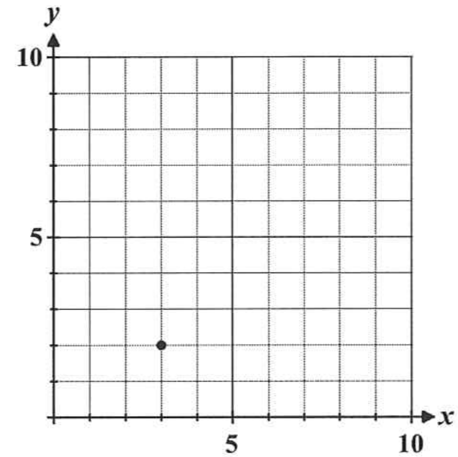
In the last lesson we began to look at **equations of proportional relationships**.

***Exercise* #1:** Let a proportional relationship between x and y exist such that the point $(3, 2)$ lies on its graph.



(a) What is the equation of this relationship? Solve this for the variable y in terms of x.

(b) Give the values of y for each of the following values of x based on your equation in (a). Show the substitution.

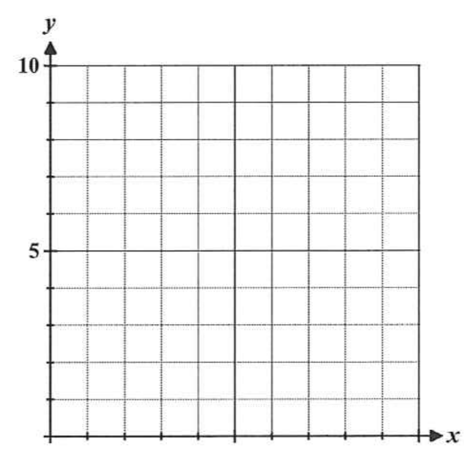
$x = 0$: $y =$ $\qquad$ $x = 6$: $y =$ $\qquad$ $x = 9$: $y =$

(c) Plot these (x, y) points on the graph. Connect with a line.

(d) State the ratio of the increase in y to the increase in x for this proportional relationship. Illustrate on the graph.

The value of $\frac{2}{3}$ in the last exercise is known as the **slope** of the line. When written in **ratio (fraction) form** the **numerator will always give the change in y** and the **denominator will give the change in x.**

***Exercise* #2:** Graph each of the following using its slope. Label with its equation.

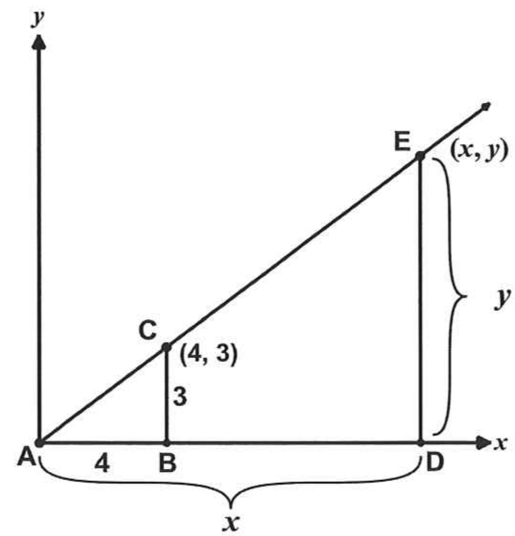


(a) $y = \frac{1}{3}x$ $\qquad$ (b) $y = \frac{3}{2}x$

(c) $y = 4x$ $\qquad$ (d) $y = \frac{3}{4}x$

A basic question remains unanswered: why do straight lines through the origin have equations of the form $y = m \cdot x$, where ***m* is the slope of the line**? Incredibly, it's due to **similar triangles**.

***Exercise* #3:** In the picture at the right, a straight line passes through the origin, at A, and through point C, at $(4, 3)$. It also passes through point E at (x, y). Vertical lines are drawn from C and E down to the x axis to points B and D.



(a) What types of angles are $\angle ABC$ and $\angle ADE$? Mark these on the diagram.

(b) Explain why $\angle ACB$ and $\angle AED$ must be congruent? (Hint: think about parallel lines).

(c) What does the information in (a) and (b) tell you about ΔABC and ΔADE? Draw them separately below and label with congruent angle pairs and dimensions.

(d) What can you now say the ratio $\dfrac{y}{x}$ will always be equal to for ΔADE no matter how large or small?

(e) Write an equation for y in terms of x.

What is the slope of this line?

(f) Another point lies on this line with coordinates of $(20, k)$. What is the value of k?

Lesson Title: ____

Date: ___________________

On a scale of 1–10, please rate the following.

I thought the lesson was explained well.

1 2 3 4 5 6 7 8 9 10

I understand what slope is.

1 2 3 4 5 6 7 8 9 10

Identify 1 <u>specific</u> thing you struggled with understanding for this lesson.

- ____
  ____

Identify 1 <u>specific</u> thing you understand but will need more practice with from this lesson.

- ____
  ____

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EQUATIONS OF LINES
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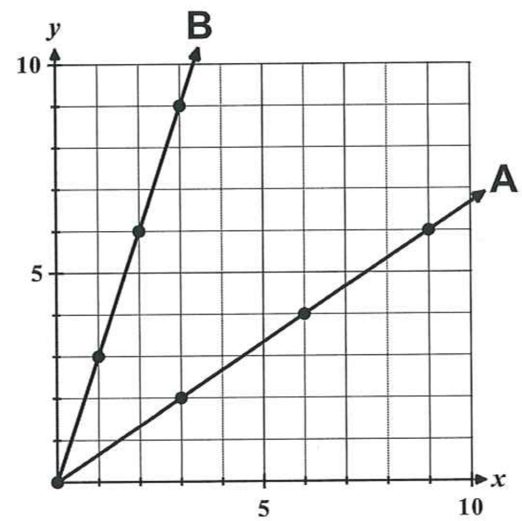
Up to this point we have graphed straight lines that **pass through the origin**. We have seen how these lines all have equations that look like $y = mx$, where the number represented by m is the **slope** of the line and measures the **change in y** divided by the **change in x.**

***Exercise* #1:** The graph below shows lines A and B. Answer the following questions.

(a) What are the slopes of both lines in ratio form? Illustrate both answers on the graph.

Slope of A: Slope of B:



(b) Which line is **steeper**? How can you tell this from comparing the values of the slopes in (a)?

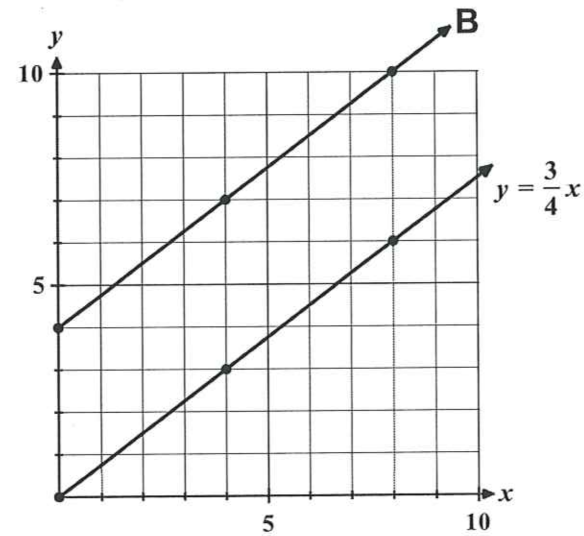
(c) What are the equations of both lines?

Equation of line A: Equation of line B:

Now we would like to start understanding equations of lines that **do not pass through the origin**.

***Exercise* #2:** The line $y = \frac{3}{4}x$ is shown graphed along with line B. Answer the following.

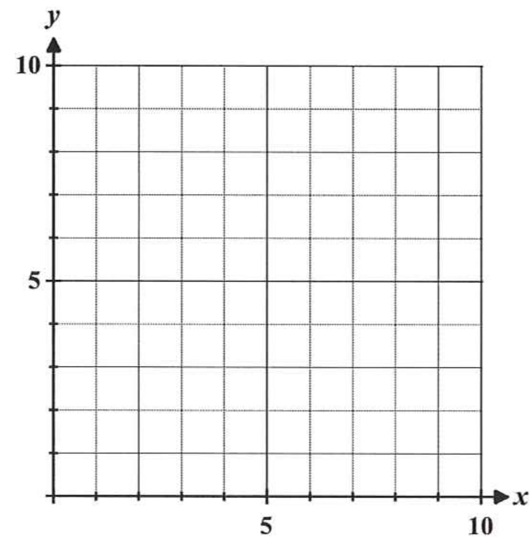
(a) Why can we say that the slope of line B is still $\frac{3}{4}$ even though it does not pass through the origin?



(b) What is true about each point on line B compared to the point on $y = \frac{3}{4}x$ at the same x-value?

We are now ready to begin graphing lines that do not pass through the origin. We will do this by generating (x, y) pairs that **satisfy** the equation (make it true).

***Exercise* #3:** Consider the equation $y = \frac{1}{2}x + 3$.



(a) Fill in the table below and then plot the points created. Connect with a straight line.

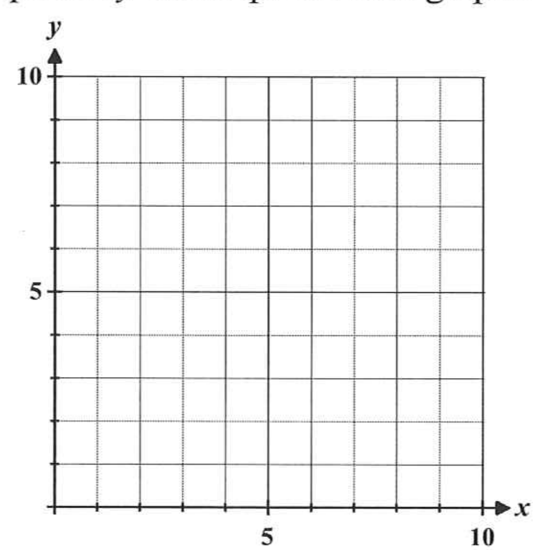
x	$\frac{1}{2}x+3$	(x, y)
0		
2		
4		
6		
8		

(b) What is the slope of this line?

(c) At what y-coordinate does the line intersect the y-axis? (The ***y*-intercept** of the line.)

When an equation is written in the form $y = mx + b$ then the m is the **slope** of the line and the b value indicates where the line "starts" on the y-axis, known as the ***y*-intercept**.

***Exercise* #4:** For each equation shown below, identify its slope and y-intercept and then graph it on the grid provided using both.



(a) $y = \frac{2}{3}x + 6$ slope: $y-$intercept:

(b) $y = 2x + 1$ slope: $y-$intercept:

Lesson Title: ____

Date: ____________________

On a scale of 1–10, please rate the following.

I thought the lesson was explained well.

1 2 3 4 5 6 7 8 9 10

I can find the equation of a line by looking at its graph.

1 2 3 4 5 6 7 8 9 10

Identify 1 <u>specific</u> thing you struggled with understanding for this lesson.

- ____
  ____

Identify 1 <u>specific</u> thing you understand but will need more practice with from this lesson.

- ____
  ____

Name: ______________________________ Date: ____________________

FINDING THE SLOPE OF A LINE
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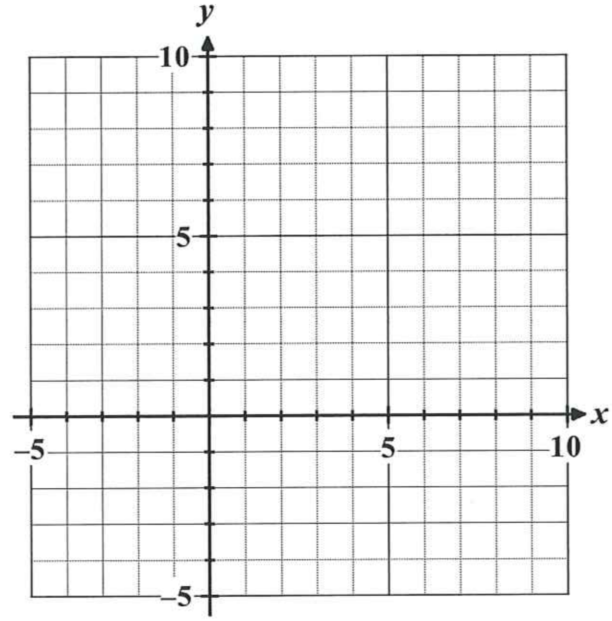
One of the most basic facts about lines is that **through any two points there can be only a single line drawn**. Another way of saying this is that **any two points uniquely determine a line**. Let's see this in the first exercise.

***Exercise* #1:** Given the points $A(2,3)$ and $B(8,6)$ do the following.

(a) Plot the two points on the grid and draw the line that passes between them.



(b) Plot other points with integer coordinates that lie on the line you drew in (a).

(c) What is the slope of the line?

$$m=\frac{\Delta y}{\Delta x}=\frac{\text{rise}}{\text{run}}=$$

(d) Write the equation of the line in $y=mx+b$ form.

We can calculate the slope of a line by just knowing the two points that fall on the line.

***Exercise* #2:** Consider the original two points in *Exercise* #1, $A(2,3)$ and $B(8,6)$.

(a) What change in x occurs from A to B?

$\Delta x=$

(b) What change in y occurs from A to B?

$\Delta y=$

(c) What calculation could you have used to find the changes in (a) and (b)? Illustrate.

(d) What is the ratio of the change in y to the change in x? What is this called?

$$\frac{\Delta y}{\Delta x}=$$

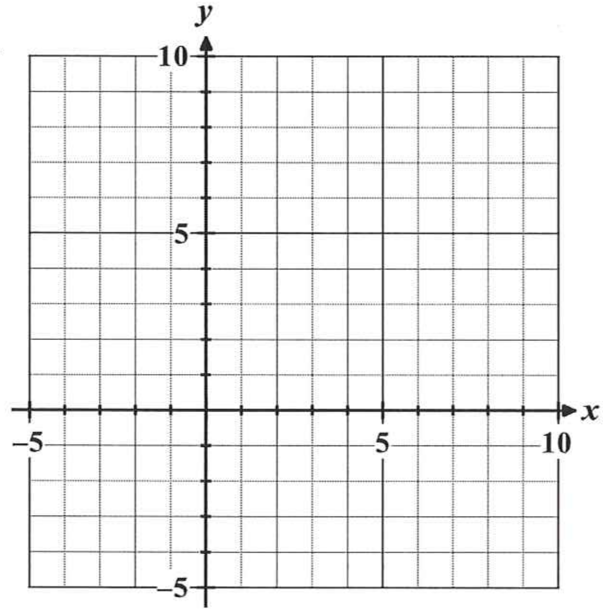
THE SLOPE FORMULA

A line that passes through the points (x_1, y_1) and (x_2, y_2) has a slope given by:

$$\text{slope} = m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

***Exercise* #3:** Consider the line that passes through the points $(-3, 2)$ and $(5, 8)$.

(a) Plot the points and draw a straight line through them.



(b) Calculate the change in x and the change in y. Show your calculations below.

$\Delta x =$ $\qquad\qquad\qquad\qquad$ $\Delta y =$

(c) Calculate the slope and express it in simplest form.

(d) Illustrate the slope you found in (c) on the grid.

We should be able to calculate the slope of a line given any two points that lie on that line. Get some practice on this essential skill below. Be careful with your positive and negative values.

***Exercise* #4:** For each set of points, calculate the change in x, change in y, and the slope of the line that passes through the points. Express your answers in simplest form.

(a) $(3, 2)$ and $(7, 12)$

(b) $(3, 5)$ and $(9, 2)$

(c) $(-6, 2)$ and $(6, -2)$

(d) $(-7, -3)$ and $(-2, 7)$

Lesson Title: ____

Date: ____________________

On a scale of 1–10, please rate the following.

I thought the lesson was explained well.

1 2 3 4 5 6 7 8 9 10

I can find the slope of a line given two points.

1 2 3 4 5 6 7 8 9 10

Identify 1 <u>specific</u> thing you struggled with understanding for this lesson.

- ____
  ____

Identify 1 <u>specific</u> thing you understand but will need more practice with from this lesson.

- ____
  ____

Name: ______________________________ Date: ______________________

AVERAGE RATE OF CHANGE
N-GEN MATH® 8

In the last lesson we looked at how the outputs of functions **increased** or **decreased** as their **input increased**. In this lesson we will investigate how **fast** this increase or decrease occurs. We generally measure this using what is known as **average rate of change**.

***Exercise* #1:** The number of people who have entered a concert hall was measured as a function of time since the doors opened. The data is shown in the table below.

Time (minutes)	0	4	10	15	18	25
People	0	48	144	239	275	338

(a) At what rate, in people per minute, did the hall fill up in the first four minutes?

(b) How many people entered the hall between 10 and 15 minutes?

(c) At what rate did people enter the hall, in people per minute, between 10 and 15 minutes?

(d) Was the rate in (c) greater or less than the rate that people entered the hall between 15 and 18 minutes?

To calculate the rate that the function is changing over any given **interval** of the **input** we simply divide the change in the function by the change in its input.

AVERAGE RATE OF CHANGE

The average rate of change can be calculated by $\dfrac{\text{change in the output}}{\text{change in the input}} = \dfrac{\Delta y}{\Delta x}$

***Exercise* #2:** The formula to calculate the average rate of change of a function is identical to what formula that you have already learned?

Average rate of change finds the **slope** between any two points on the function. The **slope** has always measured how fast the y variable is changing compared to the x variable.

***Exercise* #3:** A ball is thrown into the air such that its height is a function of time since it was thrown. The data is given in the table to the right.

Time (sec)	Height (ft)
0	5
1	91
2	145
3	167
4	157
5	115
6	41

(a) Find the average rate of change from 1 to 3 seconds in feet per second. Show your calculation.

(b) Find the average rate of change from 3 to 6 seconds in feet per second. Show your calculation.

(c) Why is the average rate of change positive in (a) but negative in (b)?

So far, we have been looking at average rate of change using tables, which is very convenient because input and output values are easy to read. But, we can also find average rate of change when functions are represented in other forms.

***Exercise* #4:** Two functions are shown below. Determine which function has the greater average rate of change from $x=-2$ to $x=4$. Show the calculations that lead to your result.

Function A: $y=10x+3$ **Function B:** $y=3x^2$

Lesson Title: ______________________________________

Date: ________________

On a scale of 1–10, please rate the following.

I thought the lesson was explained well.

1 2 3 4 5 6 7 8 9 10

I can find the average rate of change of a function.

1 2 3 4 5 6 7 8 9 10

I understand that "slope" is another word for "average rate of change".

1 2 3 4 5 6 7 8 9 10

Identify 1 <u>specific</u> thing you struggled with understanding for this lesson.

- ____
  ____

Identify 1 <u>specific</u> thing you understand but will need more practice with from this lesson.

- ____
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LINEAR FUNCTIONS
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There are many different types of functions. **Linear functions**, ones whose graphs form **lines**, are one of the most important. In this lesson we will begin to understand how to work with them.

***Exercise* #1:** Consider the **linear function** $y=\frac{3}{2}x+2$. Answer the following questions.

(a) Find the average rate of change for each of the following intervals. Show your work.

from $x=0$ to $x=6$ from $x=-4$ to $x=4$

(b) What important characteristic of the line do both of your answers from (a) equal?

THE DEFINING CHARACTERISTIC OF LINEAR FUNCTIONS

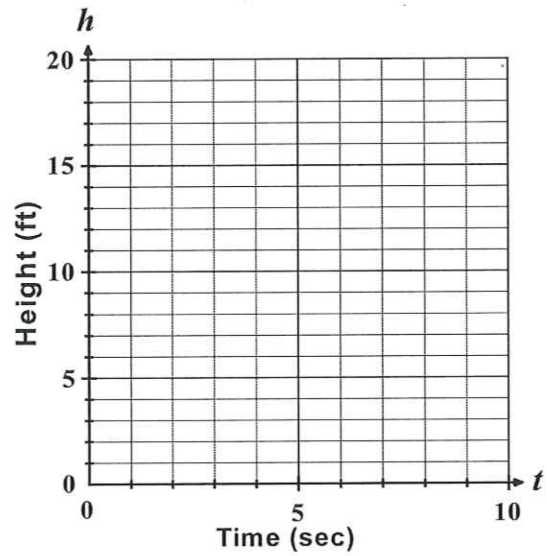
Linear functions have **constant rates of change** equal to their **slopes**. Any two points on a linear function will allow you to calculate its rate of change.

Almost any situation with a **constant rate of change** can be modeled with a linear function.

***Exercise* #2:** A balloon is let go at a height of 4 feet above the ground and rises at a (constant) rate of 2 feet per second.

(a) Fill in the table below for the height of the balloon.

t (sec)	0	1	2	3	4	5
h (ft)						



(b) Graph the data from the table and connect with a line.

(c) What are the slope and y-intercept of this line?

$m=\text{slope}=$ __________ $b=y-\text{int}=$ __________

If we can identify the **slope** and ***y*-intercept** of a linear function then we can write its equation.

***Exercise* #3:** In the last exercise our input variable was time, t, and our output variable was height, h. Given your answers to *Exercise* #2 (c), write the equation of the line.

(a) in $y = mx + b$ form

(b) in $h = mt + b$ form

If we are given the **rate** (slope) and **initial value of the output** (y-intercept) then we can always write the equation of the linear function.

EQUATIONS OF LINEAR FUNCTIONS

The equation of all linear functions can be summarized by: $y = (\text{rate}) \cdot x + (\text{initial value})$.

***Exercise* #4:** For each of the following scenarios, identify a rate and an initial value and write the linear equation that models the scenario.

(a) Lizzie has \$50 in her bank account. She decides to save an additional \$15 per week of her allowance. If she doesn't spend any more money, write a linear function for the amount of money she has saved, a, as a function of the number of weeks, w, she has been saving.

$\text{slope} = \text{rate} = m =$ __________ $y - \text{intercept} = \text{initial value} =$ __________

equation: ________________________________

(b) An elevator begins 120 feet above the ground floor. Its height **decreases** at a rate of 4 feet per second. Write a linear function for the height, h, of the elevator above the ground floor as a function of the time, t, in seconds it has been moving downward.

$\text{slope} = \text{rate} = m =$ __________ $y - \text{intercept} = \text{initial value} =$ __________

equation: ________________________________

(c) A container weighs 6.5 pounds and is filled with bricks that weigh 1.5 pounds per brick. Write a linear function for the total weight of the container, w, as a function of the number of bricks, n, that have been placed in it.

$\text{slope} = \text{rate} = m =$ __________ $y - \text{intercept} = \text{initial value} =$ __________

equation: ________________________________

Lesson Title: ____

Date: ________________

On a scale of 1–10, please rate the following.

I thought the lesson was explained well.

1 2 3 4 5 6 7 8 9 10

I can write the equation of a linear function.

1 2 3 4 5 6 7 8 9 10

I understand that linear functions have a constant rate of change.

1 2 3 4 5 6 7 8 9 10

Identify 1 <u>specific</u> thing you struggled with understanding for this lesson.

- ____
  ____

Identify 1 <u>specific</u> thing you understand but will need more practice with from this lesson.

- ____
  ____