Math in Focus™: Theory and Practice

Math in Focus is more than a curriculum, it is a coherent unfolding of mathematics based upon a proven pedagogy that has been successful in Singapore for 20 years!

When one begins a new curriculum, there are so many different (and sometimes overwhelming) factors that take time and consideration that sometimes one loses focus on what makes a program worthwhile and successful.

This newsletter contains a few aspects of Singaporean teaching to consider as you move through your implementation. These are topics that lead our group's discussions and that our own group of teachers of *Math in Focus* considers in teaching lessons each day.

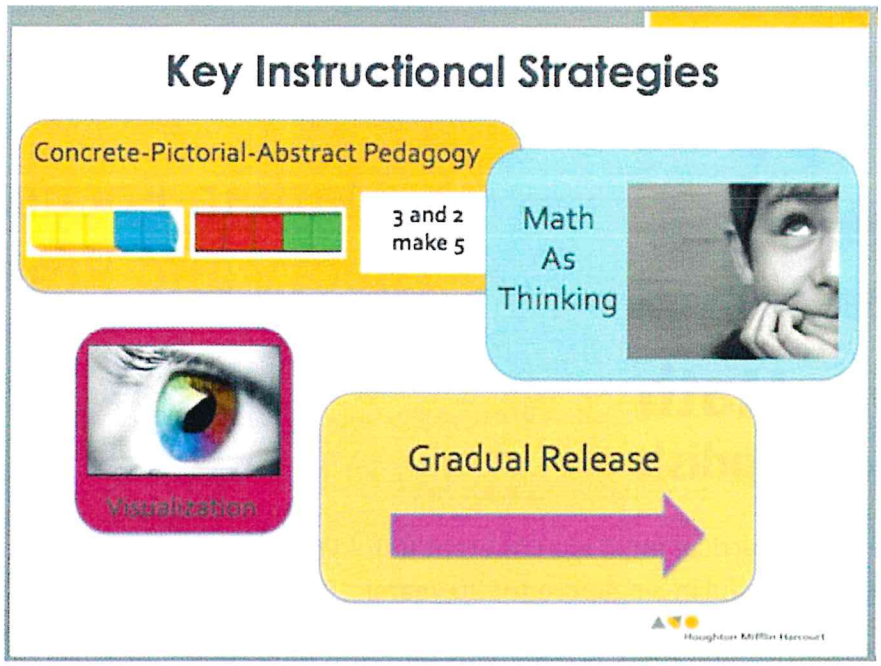
Math in Focus Non-Negotiables:

What We Believe/What We Say

Here are a few things to keep in mind:

Instructional Strategies

Below are the non-negotiable instructional strategies that should guide all lessons. Since students' books are closed during the **Teach/Learn** and even some of the **Guided Practice** sections, the instruction in the book that says, "point to the picture in the book" should be overridden by these key instructional strategies that make the lessons Singaporean. As a *Math in Focus* Professional Development Group, we are trying to change instructional practices in the classroom. Adopting *Math in Focus* is not just about adopting a new textbook series, but adopting a focused/coherent curriculum and using Singaporean instructional strategies to change the way the curriculum is delivered to students, and in turn, positively affect the success of students in these classrooms.



Concrete-Pictorial-Abstract

All concepts begin in the concrete stage. If later grade levels begin with the pictorial or abstract stage and the students don't understand the content, one must go back to the concrete stage, not give practice problem after practice problem to try to teach the concept.

In the concrete stage, the **teacher** begins instruction by modeling each mathematical concept with concrete materials and **students represent problems with concrete materials**. The importance of students taking over the responsibility of creating mathematical situations cannot be emphasized enough.

In the **pictorial** stage, the **teacher and student** transform the concrete model into a **pictorial** level, which may involve drawing pictures using circles, dots, rectangles, tallies, etc. Initially, the picture looks exactly like the concrete material then becomes more abstract. The students will recognize a picture before being able to create the picture. But, until the student can create the mathematical situation pictorially, true understanding will not happen. Students do not learn/understand/use visuals to understand visuals—they build/create through concrete experiences to draw and understand visuals. One doesn't teach the visual—the visual happens as a result of the concrete.

At the **abstract** stage, teachers and students record the **mathematics concept at a symbolic level**, using only numbers, notation, and mathematical symbols to represent the concept. Operation symbols to indicate addition, subtraction, multiplication or division and symbolic representations of amounts are used. At this level, true understanding of concepts can be seen.

Implementing a concrete-pictorial-abstract progression into a classroom can, at times, not run as smoothly as intended nor yield the results one wants. Unless the student has moved through each phase, true understanding is not developed. Even a student who demonstrates math content at the abstract level may not actually have understanding. Examples of this lack of understanding include a student who can count on

to add, but would count 9 more in the problem 3 + 9; a student who can use the algorithm to solve a 3-digit subtraction problem but does not understand why using it to solve 300 – 299 is not efficient; a student who can solve long division problems, but could not show/explain it with concrete materials.

In theory, we can understand the importance of moving through each phase and would take the time to give students these experiences, but in practice there can be roadblocks. Here are a couple of questions teachers have raised regarding the CPA Pedagogy and suggestions for moving forward.

I taught with the manipulatives, but my students still don't show understanding OR how long do I allow them to use the manipulatives?

Developing conceptual understanding is an individual process. Some students might be able to represent situations easily with concrete materials and then move on to drawing those same relationships, some may be able to represent the situations with concrete materials, but can only recognize similar drawings instead of creating them him/herself, and others may study and use the manipulatives until you give them a reason to move forward. The important thing to keep in mind is that each stage is a necessary part of building understanding. If one skips the pictorial and tries to tie concrete materials to the abstract, students may initially show understanding, but will not have the foundation for applying the understanding later. Equally, only using teacher drawings or pictorial representations from the book, will not prove that the students have created their own visual and thus leave the abstract representation weak. Allow students to use concrete materials until they can create the pictorial representation successfully. And remember, the journey to understanding is not linear so students may waver between stages.

What about my students who already know the abstract rule? How will I know whether they really understand?

One time, when showing a long division lesson from grade 4, a teacher said, "Oh, my students already know how to divide, if I show them this, it will totally mess them up!" It was only a spit second before she realized what that meant and she said, "Wow! I need to back up!" Teachers of intermediate grades, when adopting *Math in Focus* and its pedagogy, struggle with the difficulty and rigor of the new standards. However, they have the added difficulty of asking students to prove their answers concretely or pictorially when all the students know is the procedure. The students can usually get to the answer, but have no understanding of how or why it is the answer and that doesn't seem to bother them.

This procedural knowledge typically got them through elementary school and didn't cause a problem until they were presented with non-routine problem solving situations and algebraic reasoning. This will no longer be the case with the new standards.

It is imperative that you ask students to build, create, draw and/or explain for all situations even if the book doesn't ask them to (remember, it is written on the assumption that the understanding has already been built). It is not necessary to have them do it for every problem perhaps (if they have the understanding behind it), but a few on each assignment will serve as a record of their understanding. Use the **Transition Guide** to find the grade level that the understandings are developed concretely and pictorially, and introduce the students to some of those lessons/games/hands-on activities to begin the understanding.

Math is Thinking

All lessons should make students think. Each day, the teacher has the opportunity to launch the lesson in a procedural way or as a task that helps develop understanding. If students can readily answer the teacher's question, then it wasn't a problem-solving situation. Students should leave a lesson tired from thinking, but inspired to learn more!

Gradual Release

Learning a new concept takes time. Most lessons are multiple days so that students have time to assimilate new learning into their schema. The lesson is not the teacher hitting exact parts of the lesson, it's about kids walking away challenged and thinking about things. That's the reason there are multiple days to build concrete to pictorial to abstract...because kids have to make sense of something they didn't know before.

Questioning

Asking questions is what brings the lessons to life. Each lesson should have questions that the teacher has carefully thought out to help build student understanding. Questions help differentiate, should support, and should challenge. Some things to consider:

- Through questioning, all students can show what they understand
- It's not about the question; it's about the discussion that follows
- So what kind of questions are you asking?

1. How do you know?
2. Are you sure?
3. How could you check our answer?
4. Is there another way?
5. Did your classmates get the same answer?
6. How do you know which answer is correct?
7. Can there be two correct answers?
8. And many more...